OCR
RECOGNISING ACHIEVEMENT

Monday 25 June 2012 – Afternoon

A2 GCE MATHEMATICS

4731 Mechanics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4731
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 A uniform square lamina, of mass 4.5 kg and side 0.6 m, is rotating about a fixed vertical axis which is perpendicular to the lamina and passes through its centre. A stationary particle becomes attached to the lamina at one of its corners, and this causes the angular speed of the lamina to change instantaneously from $2.2\,\mathrm{rad\,s^{-1}}$ to $1.5\,\mathrm{rad\,s^{-1}}$.

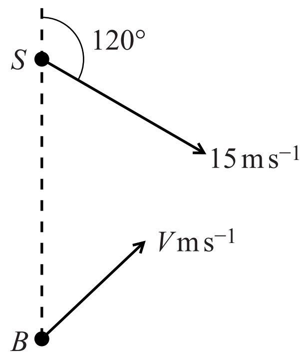
(i) Find the mass of the particle. **[4]**

The lamina then slows down with constant angular deceleration. It turns through 36 radians as its angular speed reduces from $1.5\,\mathrm{rad\,s^{-1}}$ to zero.

(ii) Find the time taken for the lamina to come to rest. **[2]**

2 A uniform solid of revolution is formed by rotating the region bounded by the x-axis and the curve $y = x\left(1 - \frac{x^2}{a^2}\right)$ for $0 \leqslant x \leqslant a$, where a is a constant, about the x-axis. Find the x-coordinate of the centre of mass of this solid. **[7]**

3



A ship S is travelling with constant velocity $15\,\mathrm{m\,s^{-1}}$ on a course with bearing 120°. A patrol boat B observes the ship when S is due north of B. The patrol boat B then moves with constant speed $V\,\mathrm{m\,s^{-1}}$ in a straight line (see diagram).

(i) Given that $V = 18$, find the bearing of the course of B such that B intercepts S. **[3]**

(ii) Given instead that $V = 9$, find the bearing of the course of B such that B passes as close as possible to S. **[5]**

(iii) Find the smallest value of V for which it is possible for B to intercept S. **[2]**

4 A uniform lamina of mass 18 kg occupies the region bounded by the x-axis, the y-axis, the line $x = \ln 9$ and the curve $y = e^{\frac{1}{2}x}$ for $0 \leqslant x \leqslant \ln 9$. The unit of length is the metre. Find the moment of inertia of this lamina about the x-axis. **[7]**

5 A uniform rod of mass 4 kg and length 2.4 m can rotate in a vertical plane about a fixed horizontal axis through one end of the rod. The rod is released from rest in a horizontal position and a frictional couple of constant moment 20 N m opposes the motion.

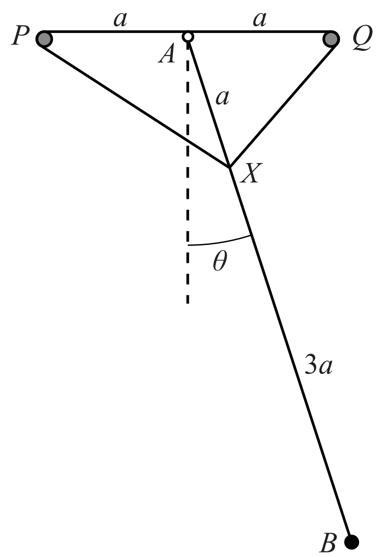
(i) Find the angular acceleration of the rod immediately after it is released. **[4]**

(ii) Find the angle that the rod makes with the horizontal when its angular acceleration is zero. **[3]**

(iii) Find the maximum angular speed of the rod. **[5]**

(iv) The rod first comes to instantaneous rest after rotating through an angle θ radians from its initial position. Find an equation for θ, and verify that $2.0 < \theta < 2.1$. **[3]**

6

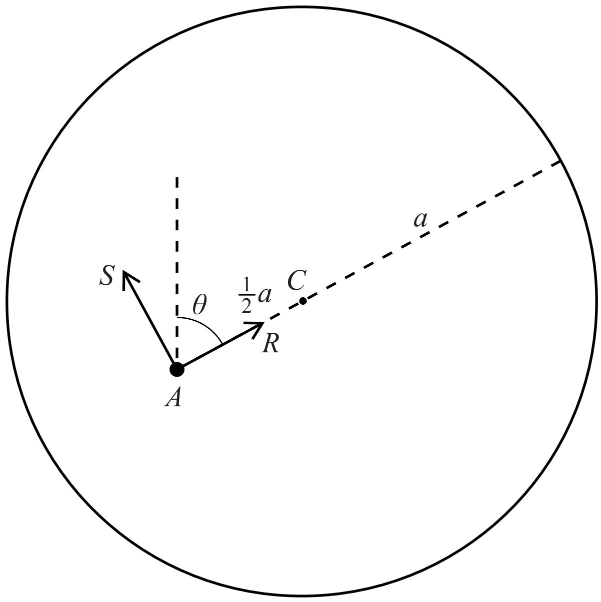


Two small smooth pegs P and Q are fixed at a distance $2a$ apart on the same horizontal level, and A is the mid-point of PQ. A light rod AB of length $4a$ is freely pivoted at A and can rotate in the vertical plane containing PQ, with B below the level of PQ. A particle of mass m is attached to the rod at B. A light elastic string, of natural length $2a$ and modulus of elasticity λ, passes round the pegs P and Q and its two ends are attached to the rod at the point X, where $AX = a$. The angle between the rod and the downward vertical is θ, where $-\frac{1}{2}\pi < \theta < \frac{1}{2}\pi$ (see diagram). You are given that the elastic energy stored in the string is $\lambda a(1 + \cos\theta)$.

(i) Show that $\theta = 0$ is a position of equilibrium, and show that the equilibrium is stable if $\lambda < 4mg$. **[6]**

(ii) Given that $\lambda = 3mg$, show that $\ddot{\theta} = -k\frac{g}{a}\sin\theta$, stating the value of the constant k. Hence find the approximate period of small oscillations of the system about the equilibrium position $\theta = 0$. **[6]**

7



A uniform circular disc with centre C has mass m and radius a. The disc is free to rotate in a vertical plane about a fixed horizontal axis passing through a point A on the disc, where $AC = \frac{1}{2}a$. The disc is slightly disturbed from rest in the position with C vertically above A. When AC makes an angle θ with the upward vertical the force exerted by the axis on the disc has components R parallel to AC and S perpendicular to AC (see diagram).

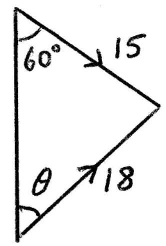
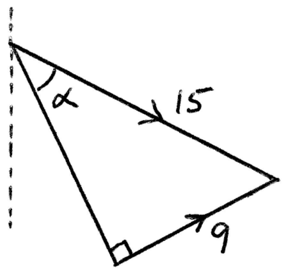
(i) Show that the angular speed of the disc is $\sqrt{\dfrac{4g(1-\cos\theta)}{3a}}$. **[4]**

(ii) Find the angular acceleration of the disc, in terms of a, g and θ. **[2]**

(iii) Find R and S, in terms of m, g and θ. **[6]**

(iv) Find the magnitude of the force exerted by the axis on the disc at an instant when $R = 0$. **[3]**

Question			Answer	Marks	Guidance	
1		(i)	$I_1 = \frac{1}{3}(4.5)(0.3^2 + 0.3^2) \quad (= 0.27)$	B1		
			$I_2 = I_1 + m(0.3^2 + 0.3^2) \quad (= 0.27 + 0.18m)$	B1		
			$I_2 \times 1.5 = I_1 \times 2.2$ $I_2 = 0.396$	M1	Using angular momentum	
			Mass is 0.7 kg	A1 **[4]**		
		(ii)	$36 = \frac{1}{2}(1.5+0)t$	M1	Using $\theta = \frac{1}{2}(\omega_0 + \omega_1)t$	*Or other complete method* ($a = -0.03125$)
			Time is 48 s	A1 **[2]**		
2			$V = \int_0^a \pi x^2 \left(1 - \frac{x^2}{a^2}\right)^2 dx$	M1	For $\int x^2 \left(1 - \frac{x^2}{a^2}\right)^2 dx$	
			$= \pi \left[\frac{x^3}{3} - \frac{2x^5}{5a^2} + \frac{x^7}{7a^4} \right]_0^a \quad (= \frac{8\pi a^3}{105})$	A1	For $\frac{x^3}{3} - \frac{2x^5}{5a^2} + \frac{x^7}{7a^4}$	
			$V\bar{x} = \int \pi x y^2 \, dx = \int_0^a \pi x^3 \left(1 - \frac{2x^2}{a^2} + \frac{x^4}{a^4}\right) dx$	M1	For $\int xy^2 \, dx$	
			$= \pi \left[\frac{x^4}{4} - \frac{x^6}{3a^2} + \frac{x^8}{8a^4} \right]_0^a \quad (= \frac{\pi a^4}{24})$	A2	For $\frac{x^4}{4} - \frac{x^6}{3a^2} + \frac{x^8}{8a^4}$	Give A1 if one error
			$\bar{x} = \frac{\frac{1}{24}\pi a^4}{\frac{8}{105}\pi a^3}$	M1	*Dependent on previous M1M1*	
			$= \frac{35a}{64}$	A1 **[7]**	*Accept* $0.547a$	

Question			Answer	Marks	Guidance	
3		(i)		B1	Velocity triangle	
			$\frac{\sin\theta}{15} = \frac{\sin 60°}{18}$	M1	*Implies previous B1*	
			Bearing is $046.2°$ (3 sf)	A1 **[3]**	*Accept* $46°$ *or* $046°$	
		(ii)		M1 A1	Relative velocity perpendicular to $\mathbf{v}_B$ Correct velocity triangle	
			$\sin\alpha = \frac{9}{15}$	M1		
			$\alpha = 36.9°$	A1	*Or other angle is* $53.1°$	
			Bearing is $30° + \alpha = 066.9°$ (3 sf)	A1 **[5]**		
		(iii)	Minimum $V = 15\sin 60°$ $= 13.0$ (3 sf)	M1 A1 **[2]**		

Question			Answer	Marks	Guidance	
4			Area is $\int_0^{\ln 9} e^{\frac{1}{2}x}\,dx$	M1	For $\int e^{\frac{1}{2}x}\,dx$	
			$=\left[2e^{\frac{1}{2}x}\right]_0^{\ln 9}=4$	A1		
			Mass per unit area is $\rho=\frac{18}{4}=4.5$	M1		
			$I=\sum\frac{1}{3}(\rho y\delta x)y^2=\frac{1}{3}\rho\int y^3\,dx$	M1	For $\int \ldots y^3\,dx$	
			$=\frac{1}{3}\rho\int_0^{\ln 9}\left(e^{\frac{1}{2}x}\right)^3 dx$	A1	Correct integral expression for I	
			$=\frac{1}{3}\rho\left[\frac{2}{3}e^{\frac{3}{2}x}\right]_0^{\ln 9}=\frac{\rho}{3}\times\frac{52}{3}$	A1	For $\int_0^{\ln 9}\left(e^{\frac{1}{2}x}\right)^3 dx=\frac{52}{3}$	
			MI is 26 kg m^2	A1 **[7]**		

Question			Answer	Marks	Guidance	
5		(i)	$I = \frac{4}{3}(4)(1.2^2) \quad (= 7.68)$	B1		
			$4 \times 9.8 \times 1.2 - 20 = I\alpha$	M1 A1	Equation of angular motion (three terms)	
			Angular acceleration is 3.52 rad s^{-2} (3 sf)	A1 **[4]**		
		(ii)	$4 \times 9.8 \times 1.2 \cos\theta - 20 = 0$ Angle is 1.13 rad (64.8°) (3 sf)	M1 A1 A1 **[3]**	Moment of weight in terms of θ	
		(iii)	WD is 20×1.132	M1 A1	Using $C\theta$ FT	
			$-20\theta = \frac{1}{2}I\omega^2 - 4 \times 9.8 \times 1.2 \sin\theta$ $-20 \times 1.132 = \frac{1}{2}(7.68)\omega^2 - 4 \times 9.8 \times 1.2 \sin(1.132)$	M1 A1	Equation involving KE $(\frac{1}{2})I\omega^2$ and PE FT if values of WD and/or θ used	
			Maximum angular speed is 2.28 rad s^{-1} (3 sf)	A1 **[5]**		
		(iv)	$20\theta = 4 \times 9.8 \times 1.2 \sin\theta$ Let $f(\theta) = 20\theta - 47.04 \sin\theta$ $f(2.0) = -2.77, \quad f(2.1) = 1.39$ Sign change shows that $2.0 < \theta < 2.1$	M1 A1 A1 **[3]**	Equation involving WD and PE AG Correctly shown	

Question			Answer	Marks	Guidance	
6		(i)	Total PE is $V = \lambda a(1+\cos\theta) - mg(4a\cos\theta)$	M1 A1	Using EPE and GPE	
			$\frac{\mathrm{d}V}{\mathrm{d}\theta} = -\lambda a\sin\theta + 4mga\sin\theta$	M1	Obtaining $\frac{\mathrm{d}V}{\mathrm{d}\theta}$	
			When $\theta = 0$, $\frac{\mathrm{d}V}{\mathrm{d}\theta} = 0$, so it is in equilibrium	B1	AG Correctly shown	*B1 for alternative proof of equilibrium (e.g. symmetry)*
			$\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = -\lambda a\cos\theta + 4mga\cos\theta$	M1	Obtaining $\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2}$	
			When $\theta = 0$, $\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} = -\lambda a + 4mga = a(4mg - \lambda)$			
			If $\lambda < 4mg$, $\frac{\mathrm{d}^2V}{\mathrm{d}\theta^2} > 0$, so equilibrium is stable	A1	AG Correctly shown	
				[6]		
		(ii)	KE is $\frac{1}{2}m(4a\dot{\theta})^2$	M1	Using KE	$\frac{1}{2}m\dot{\theta}^2$ is M0. $\frac{1}{2}I\dot{\theta}^2$ also needs attempt at $I = m(4a)^2$ for M1
			Total energy is $3mga - mga\cos\theta + 8ma^2\dot{\theta}^2 = K$	A1	FT *May still involve λ*	
			$mga\sin\theta\,\dot{\theta} + 16ma^2\dot{\theta}\ddot{\theta} = 0$	M1	Differentiating the energy equation	If wrt θ, $\frac{\mathrm{d}}{\mathrm{d}\theta}(\dot{\theta}^2) = 2\ddot{\theta}$ must be derived or clearly implied
			$\ddot{\theta} = -\frac{1}{16}\frac{g}{a}\sin\theta$ $\quad (k = \frac{1}{16})$	A1		
			$\ddot{\theta} \approx -\frac{g}{16a}\theta$, so approximate SHM	M1	Implied by $\frac{2\pi}{\omega}$ with appropriate ω	
			Approximate period is $8\pi\sqrt{\frac{a}{g}}$	A1	FT $2\pi\sqrt{\frac{a}{kg}}$	
				[6]		

Question			Answer	Marks	Guidance	
7		(i)	$I = \frac{1}{2}ma^2 + m(\frac{1}{2}a)^2 \quad (= \frac{3}{4}ma^2)$	B1		
				M1	Equation involving KE and PE	
			$\frac{1}{2}I\omega^2 = mg(\frac{1}{2}a)(1-\cos\theta)$	A1		
			$\frac{3}{8}ma^2\omega^2 = \frac{1}{2}mga(1-\cos\theta)$			
			Angular speed is $\sqrt{\dfrac{4g(1-\cos\theta)}{3a}}$	A1	AG Correctly obtained	
				[4]		
		(ii)	$mg(\frac{1}{2}a\sin\theta) = I\alpha$	M1	Equation of rotational motion	*Or differentiation of energy equation*
			$\frac{1}{2}mga\sin\theta = \frac{3}{4}ma^2\alpha$			
			Angular acceleration is $\dfrac{2g\sin\theta}{3a}$	A1		
				[2]		
		(iii)		M1	For radial acceleration $r\omega^2$	
			$mg\cos\theta - R = m(\frac{1}{2}a)\omega^2$	A1		
			$mg\cos\theta - R = \frac{2}{3}mg(1-\cos\theta)$			
			$R = \frac{1}{3}mg(5\cos\theta - 2)$	A1		
				M1	For transverse acceleration $r\alpha$	*Or* $S(\frac{1}{2}a) = I_C\,\alpha$ *(must use* I_C *)*
			$mg\sin\theta - S = m(\frac{1}{2}a)\alpha$	A1	FT if incorrect *r* already penalised	*Or* $S(\frac{1}{2}a) = (\frac{1}{2}ma^2)\alpha$
			$mg\sin\theta - S = \frac{1}{3}mg\sin\theta$			
			$S = \frac{2}{3}mg\sin\theta$	A1		
				[6]		

Question			Answer	Marks	Guidance	
		(iv)	When $R=0$, $\cos\theta=\frac{2}{5}$	M1		($\theta=1.16$ rad *or* $66.4°$)
			$\sin\theta=\frac{\sqrt{21}}{5}$, $S=\frac{2}{3}mg\left(\frac{\sqrt{21}}{5}\right)$	M1	Obtaining a value of S	
			Force exerted by axis is $\frac{2\sqrt{21}}{15}mg$	A1	Accept $0.611mg$ or $5.99m$	
				[3]		